



General Certificate of Education
Advanced Level Examination
January 2010

Mathematics

MPC4

Unit Pure Core 4

Tuesday 19 January 2010 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $f(x)$ is defined by $f(x) = 15x^3 + 19x^2 - 4$.

(a) (i) Find $f(-1)$. (1 mark)

(ii) Show that $(5x - 2)$ is a factor of $f(x)$. (2 marks)

(b) Simplify

$$\frac{15x^2 - 6x}{f(x)}$$

giving your answer in a fully factorised form. (5 marks)

2 (a) Express $\cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α , in radians, to three decimal places. (3 marks)

(b) (i) Hence write down the minimum value of $\cos x + 3 \sin x$. (1 mark)

(ii) Find the value of x in the interval $0 \leq x \leq 2\pi$ at which this minimum occurs, giving your answer, in radians, to three decimal places. (2 marks)

(c) Solve the equation $\cos x + 3 \sin x = 2$ in the interval $0 \leq x \leq 2\pi$, giving all solutions, in radians, to three decimal places. (4 marks)

3 (a) (i) Find the binomial expansion of $(1 + x)^{-\frac{1}{3}}$ up to and including the term in x^2 . (2 marks)

(ii) Hence find the binomial expansion of $\left(1 + \frac{3}{4}x\right)^{-\frac{1}{3}}$ up to and including the term in x^2 . (2 marks)

(b) Hence show that $\sqrt[3]{\frac{256}{4 + 3x}} \approx a + bx + cx^2$ for small values of x , stating the values of the constants a , b and c . (3 marks)

4 The expression $\frac{10x^2 + 8}{(x + 1)(5x - 1)}$ can be written in the form $2 + \frac{A}{x + 1} + \frac{B}{5x - 1}$, where A and B are constants.

(a) Find the values of A and B . (4 marks)

(b) Hence find $\int \frac{10x^2 + 8}{(x + 1)(5x - 1)} dx$. (4 marks)

5 A curve is defined by the equation

$$x^2 + xy = e^y$$

Find the gradient at the point $(-1, 0)$ on this curve. (5 marks)

6 (a) (i) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\sin \theta$ and $\cos \theta$. (2 marks)

(ii) Given that $0 < \theta < \frac{\pi}{2}$ and $\cos \theta = \frac{3}{5}$, show that $\sin 2\theta = \frac{24}{25}$ and find the value of $\cos 2\theta$. (2 marks)

(b) A curve has parametric equations

$$x = 3 \sin 2\theta, \quad y = 4 \cos 2\theta$$

(i) Find $\frac{dy}{dx}$ in terms of θ . (3 marks)

(ii) At the point P on the curve, $\cos \theta = \frac{3}{5}$ and $0 < \theta < \frac{\pi}{2}$. Find an equation of the tangent to the curve at the point P . (3 marks)

7 Solve the differential equation $\frac{dy}{dx} = \frac{1}{y} \cos\left(\frac{x}{3}\right)$, given that $y = 1$ when $x = \frac{\pi}{2}$.

Write your answer in the form $y^2 = f(x)$. (6 marks)

Turn over for the next question

Turn over ►

- 8 The points A , B and C have coordinates $(2, -1, -5)$, $(0, 5, -9)$ and $(9, 2, 3)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

- (a) Verify that the point B lies on the line l . (2 marks)
- (b) Find the vector \overrightarrow{BC} . (2 marks)
- (c) The point D is such that $\overrightarrow{AD} = 2\overrightarrow{BC}$.
- (i) Show that D has coordinates $(20, -7, 19)$. (2 marks)
- (ii) The point P lies on l where $\lambda = p$. The line PD is perpendicular to l . Find the value of p . (5 marks)

- 9 A botanist is investigating the rate of growth of a certain species of toadstool. She observes that a particular toadstool of this type has a height of 57 millimetres at a time 12 hours after it begins to grow.

She proposes the model $h = A\left(1 - e^{-\frac{1}{4}t}\right)$, where A is a constant, for the height h millimetres of the toadstool, t hours after it begins to grow.

- (a) Use this model to:
- (i) find the height of the toadstool when $t = 0$; (1 mark)
- (ii) show that $A = 60$, correct to two significant figures. (2 marks)
- (b) Use the model $h = 60\left(1 - e^{-\frac{1}{4}t}\right)$ to:
- (i) show that the time T hours for the toadstool to grow to a height of 48 millimetres is given by
- $$T = a \ln b$$
- where a and b are integers; (3 marks)
- (ii) show that $\frac{dh}{dt} = 15 - \frac{h}{4}$; (3 marks)
- (iii) find the height of the toadstool when it is growing at a rate of 13 millimetres per hour. (1 mark)

END OF QUESTIONS